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On the use of Markov chain models for drought class transition analysis while considering spatial effects

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Abstract

Prediction of drought class transitions has been received increasing interest in the field of water resource management. Markov chain models are effective prediction tools that are widely used to analyse drought class transitions by describing the temporal dependency of drought events. However, geophysical events or phenomena (such as drought events) can exhibit spatial effects resulting from spatial heterogeneity and/or dependency. This means that on the one hand the drought processes may vary over space, and on the other hand the state change of a drought event may not only depend on its previous state but also on the previous states of its neighbours, and it is thus unreasonable to directly apply Markov chain models without considering spatial effects. Therefore, this paper proposes a framework that considers spatial effects when employing drought class transition analysis. Three types of Markov chain models are introduced (traditional, local and spatial). To test for the existence of spatial effects, spatial clustering technology is selected to identify spatial heterogeneity, and a Q statistic is used to determine the existence of spatial dependency. Based on the results of these tests, a corresponding type of Markov chain models is then selected to analyse drought class transitions. Monthly rainfall time series data for Southwest China from 1951 to 2010 are employed in a case study, and the results show that spatial heterogeneity exists for both the 3- and 9-month SPI time series; however, the existence of spatial dependency is not confirmed. Forward and backward estimation rules are also obtained for drought class transitions using local Markov chain models.

Keywords Drought class transitions · Markov chain models · Spatial heterogeneity · Spatial dependency · Standardized precipitation index · Spatial clustering

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1 Introduction

Drought is a natural hazard that results from a deficiency in expected or ‘normal’ amounts of precipitation that, when extended over a season or longer, are not sufficient to meet the demands of human activities and the environment (Dracup et al. 1980; Whilhite 2005). Drought results in disruption to the water supply of natural ecosystems and affects normal production and daily life; therefore, prediction of drought class transitions has been received increasing interest in the field of water resource management (Panu and Sharma 2002; Nichol and Sawaid 2015).

According different theoretical backgrounds, there are two main approaches used to model drought class transitions. One is based on statistical analysis of the climatic record, which assumes that the future behaviour of drought will tend to replicate the behaviour of drought observed in the past. The other is based on mechanism analysis of the climate, which attempts to simulate the process of drought by making computer models of the climate system (Whilhite 2005). Compared to the development of statistical models, computer models representing the entire atmosphere/earth/ocean system are still in their infancy and are facing enormous challenges (Stockdale et al. 1998; Vu et al. 2015). Statistical models are still commonly used in drought class transitions, and among existing statistical models, Markov chain models are popular because they are effective for describing the probability of drought class transitions. For example, Markov chain models were used by Lohani et al. (1997, 1998) to derive drought characteristics and assess dry spells from long-term records of the Palmer Severe Drought Index (PDSI) in two climatic areas of Virginia (USA), and Banik et al. (2002) applied Markov chain models to analyse the probability of transitioning from a dry week to a non-dry or a dry week, while aiming to develop an index of drought proneness for a given area. In addition, Paulo et al. (2005) used the homogeneous and non-homogeneous formulations of Markov chain models to predict standardized precipitation index (SPI) drought class transitions, and Yang et al. (2016) applied Markov chain models to calculate the expected residence time, return period and transition probabilities of drought in the Weihe river basin.

Markov chain models generally assume that future states of drought depend only on the current state of drought but do not dependent on the historical state of drought (Isaacson and Madsen 1976; Ross 2014). These models are essentially used to describe the temporal dependency of drought from a statistical perspective. However, geographical events or phenomenon (including drought events) exhibit spatial effects resulting from spatial heterogeneity and dependency, which are the most critical concepts in the field of spatial data analysis or spatial statistics (Yang et al. 2018). Spatial heterogeneity emphasizes spatial differences that geographical events or phenomenon might vary over space rather than being constant or the distribution and relationships changes across spatial locations (Brunsdon et al. 1998; Deng et al. 2017). Spatial dependency indicates that geographical events or phenomenon close together in space tend to be more similar than those which are farther apart (Lloyd 2006), and this principle is also called as the “First Law of Geography” that everything is related to everything else, but near things are more related to each other (Tobler 1970). It is noteworthy that spatial dependency does not deny the existence of spatial heterogeneity, but it aims at describing the correlation among near different events or phenomenon.

Considering that drought is regarded as a typical spatial event or phenomenon, it is thus unreasonable to directly apply Markov chain models when analysing drought class transitions without considering spatial heterogeneity and dependency, because a global model without considering spatial heterogeneity only reflects the mean characteristic of the whole area and cannot reveal the spatial differences of drought class transitions at different sub-areas (Deng et al. 2017; Yang et al. 2017). Additionally, spatial dependency indicates that the future state change of a drought event may not only depend on its previous states, but also on the previous states of its neighbours (Cressie 1993). Obviously, without considering spatial dependency, it is difficult to identify this pattern. Therefore, this paper presents a framework for drought class transitions using Markov chain models while simultaneously considering the spatial effects resulting from spatial heterogeneity and/or dependency.

2 Methods

Figure 1 shows the analytical framework for drought class transitions using Markov chain models that simultaneously considers spatial effects. In this process, the standardized precipitation index (SPI) is firstly calculated based on monthly rainfall time series data for a given area, and it is then determined whether spatial heterogeneity exists within the SPI time series. If spatial heterogeneity does not exist, a global Markov chain model can be directly used to analyse drought class transitions. If it does exist, it is necessary to determine whether spatial dependency of the SPI drought event exists. If spatial dependency is found to exist, spatial Markov chain models are then selected to analyse drought class transitions, and if it does not exist, local Markov chain models are used. One can find that the proposed framework is a complete process to consider spatial heterogeneity and

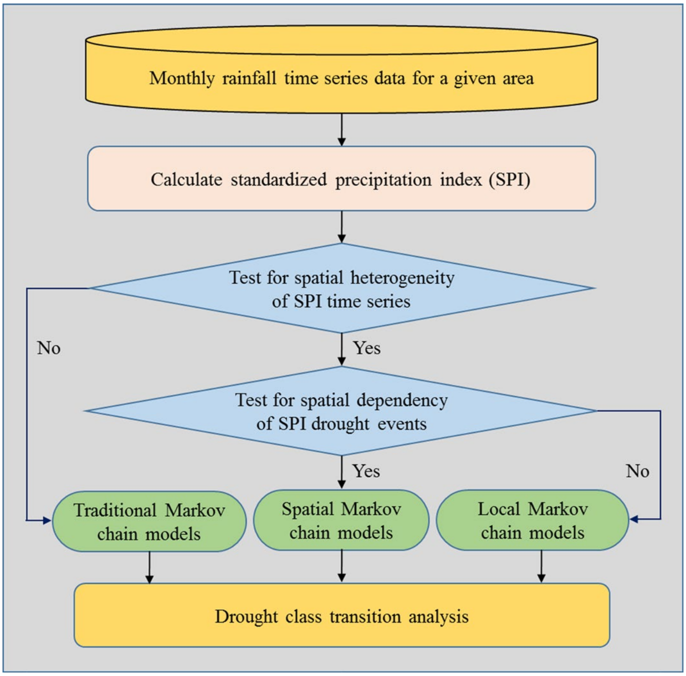

Fig. 1 Analytical framework of Markov chain models for drought class transition analysis

dependency in analysing drought class transitions, but spatial Markov chain is just a model which can be applied to handle spatial heterogeneity.

2.1 SPI drought index

The standardized precipitation index (SPI) is a normalized index that presents the probability of an observed rainfall amount occurring when compared to the rainfall climatology over the long-term at a particular geographical location (Mckee et al. 1993). It can be used to represent the probability of abnormal wetness and dryness. Positive (negative) SPI values represent a precipitation surplus (deficit). According to the different SPI values in Table 1, drought events can be divided into four classes: non-drought ($0 \leq \text{SPI}$), mild drought ($-1.0 \leq \text{SPI} < 0$), moderate drought ($-1.5 \leq \text{SPI} < -1.0$), and severe or extreme drought ($\text{SPI} < -1.5$). It can be seen that increasingly negative SPI values indicate an increasingly serious event. In addition, SPI values can be calculated for different time scales (e.g. 3-, 6-, 9-, or 12-months) and can thus facilitate analysis on the impact of drought events on various types of water resource management. For instance, soil moisture conditions respond to precipitation anomalies on a relatively short-scale term, whereas groundwater, stream and reservoir storage reflect longer-term precipitation anomalies (Moreira et al. 2008; Paulo and Pereira 2007).

Compared with physically-based PSDI, which employs a simple balance model, SPI has the following advantages: (a) only monthly precipitation data need to be collected to enable computation; (b) standardization of the SPI can be used to determine the rarity of a current drought; (c) the SPI value can be compared across areas that have markedly different climates; (d) the SPI value can be determined on different time scales (Trenberth et al. 2014). The Lincoln Declaration on Drought Indices, which was approved by the World Meteorological Organization in 2009, recommended that 'the Standardized Precipitation Index be used to characterize meteorological droughts around the world' (Hayes et al. 2011). Therefore, the SPI was selected to identify drought classes in this research.

2.2 Three types of Markov chain models

A traditional Markov chain is a stochastic process, X, and at any time, t, the probability that X_{t+1} takes a particular value, j, depends on the value of the current state, X_t, and is conditionally independent from historical states $X_{t-1},\cdots,X_0$ (Cinlar 1975). This can be represented as

$$P\{X_{t+1} = j | X_t, \ldots, X_0\} = P\{X_{t+1} | X_t = i\} \quad \forall i,j \in S,\ t \in T \tag{1}$$

A traditional Markov chain can be characterized by a set of states, S, and the transition probability, p_{ij} of a drought prediction between states i and j. The transition

Table 1 Drought class classification of SPI

Code	Drought classes	SPI values
1	Non-drought	$0 \leq \text{SPI}$
2	Mild drought	$-1.0 \leq \text{SPI} < 0$
3	Moderate drought	$-1.5 \leq \text{SPI} < -1.0$
4	Severe/extreme drought	$\text{SPI} < -1.5$

probability, p_{ij}, represents the probability that the traditional Markov chain is in state j at the next time point, when the state is i at the present time point. The transition probability matrix $P = [p_{ij}] = P\{X_{t+1} = j|X_t = i\}$ can be estimated by counting the number of times, n_{ij}, that a state series passes from state i to state j, and the estimated value can be described as

$$\hat{p}_{ij} = \frac{n_{ij}}{\sum_j n_{ij}} \tag{2}$$

The traditional Markov chain model is an effective tool for dealing with the temporal dependency of drought events, and it is essentially a global model that has constant parameters over the whole study area. As previously mentioned, the process of a drought event changes maybe over space and is not constant, and is therefore spatially heterogeneous. The global Markov chain model is unable to adequately reveal spatial variations in drought class transitions for the heterogeneous study area, and it is thus necessary to use local Markov chain models to analyse drought class transitions, whose parameters are dependent on spatial locations or areas. The probability condition of a local Markov chain model can be represented as

$$P\{X_{t+1}^{\mathrm{loc}} = j|X_t^{\mathrm{loc}}, \ldots, X_0^{\mathrm{loc}}\} = P\{X_{t+1}^{\mathrm{loc}}|X_t^{\mathrm{loc}} = i\} \quad \forall i,j \in S,\ t \in T \tag{3}$$

where loc represents spatial location information. The transition probability matrix $P^{\mathrm{loc}} = [p_{ij}^{\mathrm{loc}}] = P\{X_{t+1}^{\mathrm{loc}} = j|X_t^{\mathrm{loc}}(\mathrm{loc}) = i\}$ can be estimated by first counting the number of times, n_{ij}^{loc}, that a state series passes from state i to state j at a spatial location or area, loc, which can be written as

$$\hat{p}_{ij}^{\mathrm{loc}} = \frac{n_{ij}^{\mathrm{loc}}}{\sum_j n_{ij}^{\mathrm{loc}}} \tag{4}$$

However, local Markov chain models can only be applied to deal with spatial heterogeneity, and when drought events not only exhibit spatial heterogeneity but also show spatial dependency, an extended model of a local Markov chain model, a spatial Markov chain model that incorporates spatial dependency needs to be employed.

Spatial Markov chain models were first presented to identify whether overlapping trends of regional development existed within a regional economy (Rey 2001). To detect the overall convergence of regional income, a spatial Markov matrix was defined based on modification of the traditional Markov matrix that conditions a region's transition probabilities on the initial income class of its 'spatial context'. The 'spatial context' is defined as the neighbouring state of a region and is used to analyse spatial dependency. As illustrated in Table 2 for a case where k is equal to 3, the spatial Markov matrix decomposes the local $k \times k$ 2-dimensional transition matrix into a $k \times k \times k$ 3-dimensional transition matrix using the states of spatial context.

In Table 2, the conditional transition probability, $p_{ij}^{\mathrm{loc}}(l)$, indicates the transition probability that a spatial location or area, loc, is in state j at the next time point, given that at the present time it is in state i and its spatial context is in state l. The equation for estimating the value of $p_{ij}^{\mathrm{loc}}(l)$ can be represented as

$$\hat{p}_{ij}^{\mathrm{loc}}(l) = \frac{n_{ij}^{\mathrm{loc}}(l)}{\sum_j n_{ij}^{\mathrm{loc}}(l)} \tag{5}$$

Table 2 Spatial and local Markov transition matrices

Spatial context	$(t+1)/t$	1	2	3
1	1	$p_{11}^{\mathrm{loc}}(1)$	$p_{12}^{\mathrm{loc}}(1)$	$p_{13}^{\mathrm{loc}}(1)$
	2	$p_{21}^{\mathrm{loc}}(1)$	$p_{22}^{\mathrm{loc}}(1)$	$p_{23}^{\mathrm{loc}}(1)$
	3	$p_{31}^{\mathrm{loc}}(1)$	$p_{32}^{\mathrm{loc}}(1)$	$p_{33}^{\mathrm{loc}}(1)$
2	1	$p_{32}^{\mathrm{loc}}(2)$	$p_{12}^{\mathrm{loc}}(2)$	$p_{13}^{\mathrm{loc}}(2)$
	2	$p_{21}^{\mathrm{loc}}(2)$	$p_{22}^{\mathrm{loc}}(2)$	$p_{23}^{\mathrm{loc}}(2)$
	3	$p_{31}^{\mathrm{loc}}(2)$	$p_{32}^{\mathrm{loc}}(2)$	$p_{33}^{\mathrm{loc}}(2)$
3	1	$p_{11}^{\mathrm{loc}}(3)$	$p_{12}^{\mathrm{loc}}(3)$	$p_{13}^{\mathrm{loc}}(3)$
	2	$p_{21}^{\mathrm{loc}}(3)$	$p_{22}^{\mathrm{loc}}(3)$	$p_{23}^{\mathrm{loc}}(3)$
	3	$p_{31}^{\mathrm{loc}}(3)$	$p_{32}^{\mathrm{loc}}(3)$	$p_{33}^{\mathrm{loc}}(3)$
–	1	p_{11}^{loc}	p_{12}^{loc}	p_{13}^{loc}
	2	p_{21}^{loc}	p_{22}^{loc}	p_{23}^{loc}
	3	p_{31}^{loc}	p_{32}^{loc}	p_{33}^{loc}

$p_{ij}(k)$ indicates the probability of transfer from state i to state j when spatial context belongs to state k

$p_{ij} = \sum_{l=1}^{k} p_{ij}(l)p(l)$; p_{ij} indicates the probability of transfer from state i to state j under all conditions and the probability is $p(l)$ when spatial context belongs to state l

where $n_{ij}^{\mathrm{loc}}(l)\,(l = 1, \ldots, k)$ represents the counts that when area, loc, and its spatial context are in states i and l at the present time, respectively, area loc is in state j at the next time point.

Table 3 lists the results of a comparison between the three types of Markov chain models. Traditional Markov chain models generally assume that drought events are spatially stationary and independent, and a global model that only describes temporal dependency is thus appropriate for a homogeneous study area. Both local and spatial Markov chain models can be applied in a heterogeneous area. If it is determined that the drought events in a study area are spatially dependent, then a spatial Markov chain model that incorporates spatial context states can be reasonably selected as the analytical model, otherwise local Markov chain models are required.

Table 3 Comparison between three types of Markov chain models

Model type	Applied area	Spatial heterogeneity	Spatial dependency
Traditional Markov chain models	Global	X	X
Local Markov chain models	Local	√	X
Spatial Markov chain models	Local	√	√

The symbol √(X) indicates the model can (cannot) handle the corresponding characteristic

2.3 Test for spatial heterogeneity of SPI drought events

In spatial analysis, a heterogeneous area can be divided into a series of homogenous or quasi-homogenous subareas (Wang et al. 2016), and based on this assumption, testing for spatial heterogeneity of the SPI time series can be converted into a homogenous partition problem. In this respect, we first assume that the study area can be divided into a series of subareas based on the similarity of SPI time series, and we then evaluate the validity of the partitioning result. An evaluation result that shows the study area can be divided shows that drought events are spatially heterogeneous, and vice versa.

Spatial clustering is a widely-used strategy employed with the homogenous partition problem, and it aims to group spatial data into several meaningful clusters according to their similarity in spatial and temporal domains (Deng et al. 2017). In this respect, time series that are in the same cluster are more similar to each other than to those in other clusters, and they are more similar when they are located adjacent to each other. Spatial clustering can be used to divide the whole study area into several homogenous subareas, and spatio-temporal series at the same subareas are with the similar evolution characteristics. Therefore, for SPI-based drought analysis, the SPI spatial clustering algorithm can be used to partition the study area into several homogenous subareas.

Although a variety of spatial clustering algorithms have been developed to handle spatial data, a hierarchical clustering method known as Regionalization with Dynamically Constrained Agglomerative Clustering and Partitioning (REDCAP) was adopted in this study, as this guarantees spatial proximity and attributes similarity within clusters. The REDCAP algorithm is generally implemented in two steps: A hierarchical clustering strategy is firstly employed to generate a spatially contiguous tree, and average linkage is used to define the similarity of two clusters. The spatially contiguous tree is then partitioned into several subtrees by optimizing an objective function. A detailed introduction to this process can be found in Guo (2008).

When using the REDCAP algorithm, spatial clustering results with different numbers of clusters or subareas can be obtained. A clustering validity index, namely the Sil index, was selected to identify the validity of the partitioning result in this research. The Sil index can be used to measure the over silhouette width, which enables 'clear-cut' clusters to be distinguished from 'weak' clusters. A clustering result that corresponds to a larger Sil index is more pronounced. Therefore, if the Sil index monotonically decreases as the numbers of clusters increase, then the study area is considered to be homogenous and does not need to be divided; otherwise, spatial heterogeneity is present, and several homogenous or quasi-homogenous subareas can be obtained based on the clustering validity evaluation result.

2.4 Test for spatial dependency of SPI drought events

Spatial dependency can be described as the first law of geography where 'everything is related to everything else, but near things are more related than distant things' (Tobler 1970); this implies that the state change of drought events in a homogeneous area may also depend on the previous state of its spatial neighbours (which are defined as being the spatial context). If we assume that there are n neighbours for a homogeneous area, loc, then the value of its spatial context, $\mathrm{SPI}_{\mathrm{SC}}^{\mathrm{loc}}(t)$, can be estimated by the following formula

$$\mathrm{SPI}_{\mathrm{SC}}^{\mathrm{loc}}(t)=\frac{\sum_{j=1}^{n} w_j^{\mathrm{loc}}\mathrm{SPI}_j^{\mathrm{loc}}(t)}{\sum_{j=1}^{n} w_j^{\mathrm{loc}}} \tag{6}$$

where $\mathrm{SPI}_j^{\mathrm{loc}}(t)$ indicates the SPI value of the jth ($j = 1, \dots, n$) neighbour of the area, loc, at time, t; w_j^{loc} indicates the weight of the jth neighbouring area, which is used to reflect the strength of spatial dependency between the area, loc, and its jth neighbouring area. In this study, we assumed that the weight was obtained by the correlation coefficient of SPI time series, which meant that when the correlation coefficient was larger, the weight assigned to the neighbouring area was also larger.

If the spatial context is not important for class transition probabilities in an homogeneous area, loc, then the null hypothesis can be described as

$$p_{ij}^{\mathrm{loc}}(1)=p_{ij}^{\mathrm{loc}}(2)=p_{ij}^{\mathrm{loc}}(3)=\dots=p_{ij}^{\mathrm{loc}}(k)=p_{ij}^{\mathrm{loc}}, \quad \forall i,j \tag{7}$$

The likelihood ratio statistic, Q, (Anderson and Goodman 1957; Kullback et al. 1962) can be used to test the null hypothesis, the form of which is described as

$$Q=-2\log\left\{\prod_{l=1}^{k}\prod_{i=1}^{k}\prod_{j=1}^{k}\left[\frac{p_{ij}^{\mathrm{loc}}}{p_{ij}^{\mathrm{loc}}(l)}\right]^{n_{ij}^{\mathrm{loc}}(l)}\right\} \tag{8}$$

where k indicates the number of the drought class; p_{ij}^{loc} indicates the transition probability without considering the spatial context; $p_{ij}^{\mathrm{loc}}(l)$ and $n_{ij}^{\mathrm{loc}}(l)$ indicate spatial transition probability and numbers when the spatial context is in state i, respectively; and the statistic Q is asymptotically distributed as χ with $K(K-1)^2$ degrees of freedom.

3 Case study

The study area, southwest China, is one of the most significant grain production regions (Jia et al. 2018), and in recent decades, drought hazards have occurred frequently in this region which resulting in large agricultural losses (Zuo et al. 2014). Therefore, agricultural security and ecological restoration call for a detailed understanding of spatial effects of drought transition rules in this area. The study area comprises Chongqing, Guizhou, Sichuan and Yunan provinces in Southwest China and covers a total area of 1.1×10^6 km^2. Precipitation data for the time period (January 1951 to December 2010) were collected at 80 meteorological stations (China Meteorological Data Service Center, https://data.cma.cn/) in the study area. The distribution of these meteorological stations is shown in Fig. 2.

Considering the scale representation and the distribution diversity of drought classes, the 3- and 9-month SPI values were first calculated by the program (SPI_SL_6.exe), which can be downloaded from the National Drought Mitigation Center (https://drought.unl.edu/AboutUs.aspx). On the basis of 3- and 9-month SPI time series, the REDCAP algorithm was applied to divide the study area into several subareas with different cluster numbers. (Results of spatial clustering analysis are shown in Fig. 3.) It was found that the Sil index values of both 3- and 9-month SPI clustering results did not monotonically decrease with an increase in the numbers of clusters, and both of their curves firstly rose and then fell,

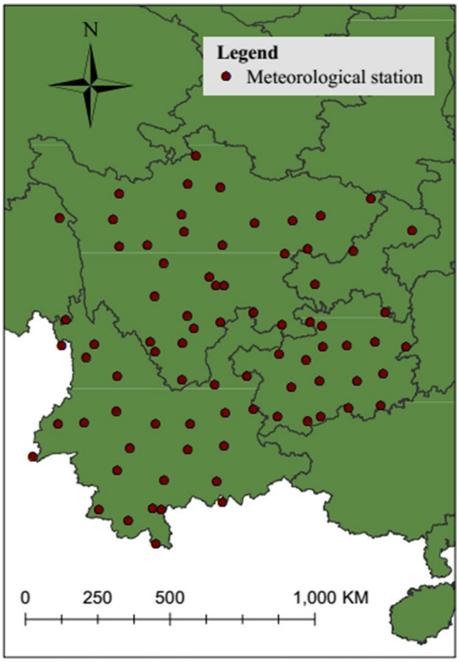

Fig. 2 Distribution of meteorological stations used in the study area

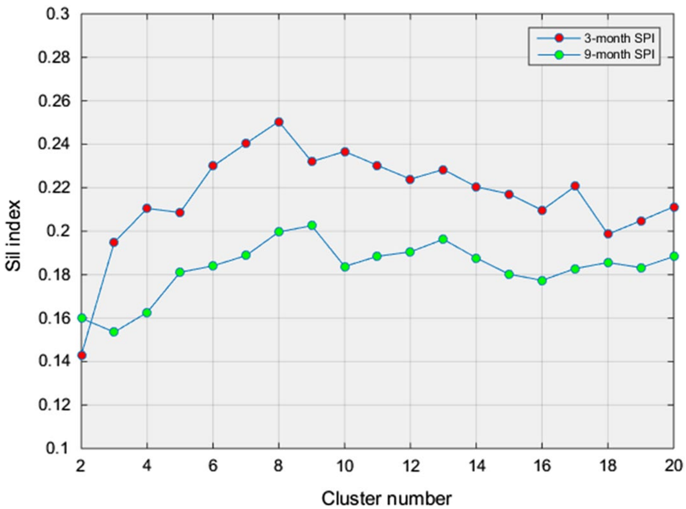

Fig. 3 Results of clustering validation evaluation for 3- and 9-month SPI time series

which indicated that spatial heterogeneity existed within the SPI time series. Furthermore, for these two time scales SPI, the study area could be divided into 8 and 10 clusters or sub-areas, respectively, which corresponded to maximum Sil index values.

The clustering results for the 3- and 9-month SPI time series are shown in Fig. 4. A comparison of the clustering results between these two scales shows that although there are certain changes in local locations, the overall results are relatively similar. As all the

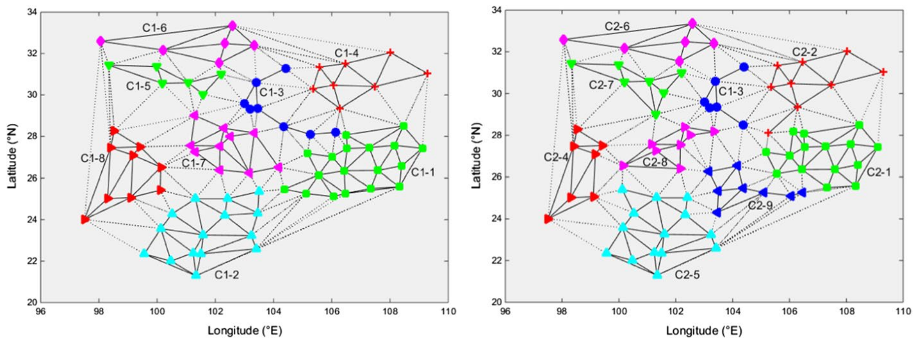

Fig. 4 Clustering results of 3- and 9-month SPI time series

neighbours of cluster C1-7 and C2-8 ({C1-1, C1-2, C1-3, C1-5, C1-8}, {C2-3, C2-4, C2-5, C2-7, C2-9}) were contained in clustering analysis results, we focused on the drought class transitions of subareas C1-7 and C2-8.

The 3- and 9-month SPI time series of subareas C1-7 and C2-8 and their spatial contexts are shown in Fig. 5. The continuous SPI values were converted into drought classes based on the information provided in Table 1. Conditional transition probabilities were computed using Eqs. (4) and (5), and the spatial and local Markov matrixes of C1-7 and C2-8 are listed in Tables 4 and 5. To test spatial dependency, the null hypothesis was that drought class transitions of subareas C1-7 and C2-8 were independent from their spatial contexts. The statistics, Q, of subareas C1-7 and C2-8 are 3.51 and 0.71, respectively. As these values are smaller than $\chi^2(36) = 51.00$ when a significance level (α) of 0.05 is selected; therefore, the null hypothesis was not rejected because it could not be proven that the state change of drought events in subareas C1-7 and C2-7 were dependent on their spatial contexts. Therefore, Markov chain models are suitable for use in analysing drought class transitions in subareas C1-7 and C2-8.

On the basis of local Markov matrixes, we defined two kinds of estimation rules for analysing drought class transitions, and these are described as follows:

(1) The forward estimation rule: If state(loc, t) = i, then state(loc, $t+1$) = j with the possibility of $p_{ij}^{\mathrm{loc}}(l)$, where state (loc, t) indicates the drought state of subarea loc at

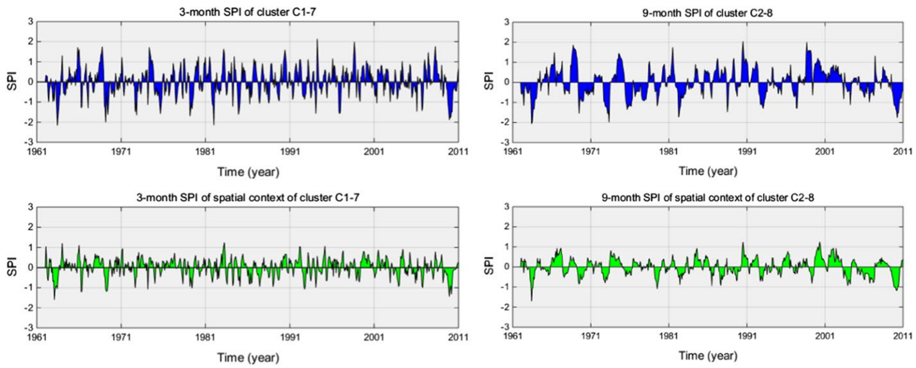

Fig. 5. 3- and 9-month SPI time series of clusters C1-7 and C2-8 and their spatial contexts

Table 4 Spatial Markov transition probability of cluster C1-7

Spatial context	$(t+1)/t$	1	2	3	4
1	1	0.6429	0.3214	0.0357	0.0000
	2	0.3333	0.5556	0.1111	0.0000
	3	0.0000	0.0000	0.0000	0.0000
	4	0.0000	0.0000	0.0000	0.0000
2	1	0.5000	0.3333	0.1667	0.0000
	2	0.4000	0.6000	0.0000	0.0000
	3	0.3333	0.0000	0.3333	0.3333
	4	0.0000	0.0000	1.0000	0.0000
3	1	0.0000	0.0000	0.0000	0.0000
	2	1.0000	0.0000	0.0000	0.0000
	3	0.5000	0.0000	0.5000	0.0000
	4	0.0000	0.0000	0.0000	0.0000
4	1	0.0000	0.0000	0.0000	0.0000
	2	0.0000	1.0000	0.0000	0.0000
	3	0.0000	0.0000	0.0000	0.0000
	4	0.0000	0.0000	0.0000	0.0000
–	1	0.6176	0.3235	0.0588	0.0000
	2	0.3846	0.5769	0.0385	0.0000
	3	0.4000	0.2000	0.2000	0.2000
	4	0.0000	0.0000	0.5000	0.5000

time t and $p_{ij}^{\mathrm{loc}}(l)$ represents forward estimation possibility. This rule can be used to estimate the next unknown state when the present state is known, which can be directly obtained from local Markov transition matrixes.

(2) Backward estimation rule: If state (loc, t) = j, then state (loc, $t-1$) = i with the possibility of $'p_{ij}^{\mathrm{loc}}$, where $'p_{ij}^{\mathrm{loc}}$ represents backward estimation possibility. This rule was can be used to estimate the previous unknown state when the present state is known, and is calculated by the following expression

$$'\hat{p}_{ij}^{\mathrm{loc}} = \frac{n_{ij}^{\mathrm{loc}}}{\sum_i n_{ij}^{\mathrm{loc}}} \tag{9}$$

According to the above defined rule, a series of specific rules can be achieved as follows:

Rule 1 If state (C1-7, t) = 1, then there is a 61.76% possibility of state(C1-7, $t+1$) = 1. This means that if the state of cluster C1-7 is non-drought at the present time point, the possibility of it continuing to be non-drought at the next time point (for drought with a 3-month time scale) is 61.76%.

Rule 2 If state(C1-7, t) = 4, then there is a 50% possibility of state(C1-7, $t+1$) = 3, which means that if the state of subarea C1-7 is severe/extreme drought at the present time point, then the possibility of the next state of subarea C1-7 being moderate drought is 61.76% (for drought with a 3-month time scale).

Table 5 Spatial Markov transition probability of cluster C2-8

Spatial context	$(t+1)/t$	1	2	3	4
1	1	0.8636	0.1364	0.0000	0.0000
	2	0.2000	0.7000	0.1000	0.0000
	3	0.0000	0.0000	0.0000	0.0000
	4	0.0000	0.0000	0.0000	0.0000
2	1	0.8333	0.1667	0.0000	0.0000
	2	0.1765	0.7647	0.0000	0.0588
	3	0.0000	0.6667	0.3333	0.0000
	4	0.0000	0.0000	0.0000	0.0000
3	1	0.0000	0.0000	0.0000	0.0000
	2	0.0000	0.0000	0.0000	0.0000
	3	0.0000	0.0000	0.0000	0.0000
	4	0.0000	0.0000	0.5000	0.5000
4	1	0.0000	0.0000	0.0000	0.0000
	2	0.0000	0.0000	0.0000	0.0000
	3	0.0000	0.0000	0.0000	0.0000
	4	0.0000	0.0000	0.0000	1.0000
–	1	0.7143	0.2857	0.0000	0.0000
	2	0.1282	0.8205	0.0513	0.0000
	3	0.200	0.2000	0.4000	0.2000
	4	0.0000	0.0000	0.5000	0.5000

Rule 3 If state(C2-8, t) = 2, then there is a 5.13% possibility of state(C2-8, $t+1$) = 3, which means that if the state of subarea C2-8 is mild drought at the present time point, then there is a 5.13% possibility of the next state of subarea C2-8 being moderate drought (for drought with 9-month time scale).
Rule 4 If state(C1-7, t) = 1, then there is a 6.06% possibility of state(C1-7, $t-1$) = 3, which means that if the present state of subarea C1-7 is non-drought, then there is a 6.06% possibility of the previous state of subarea C2-8 being moderate drought (for drought on a 3-month time scale).
Rule 5 If state(C2-8, t) = 2, then there is a 82.05% probability of state(C1-7, $t-1$) = 2, which means that if the present state of subarea C2-8 is mild drought, then there is a 82.05% possibility of the previous state of subarea C2-8 being mild drought (for drought on a 9-month time scale).
Rule 6 If state(C2-8, t) = 3, then there is a 20% probability that state(C1-7, $t-1$) = 4, which means that if the present state of subarea C2-8 is moderate drought, then there is a 20% probability of the previous state of subarea C2-8 being severe/extreme drought (for drought on a 9-month time scale).

In addition, forward and backward estimation possibility graphs can be used to express all the rules for subareas C1-7 and C2-8, as shown in Figs. 6 and 7.

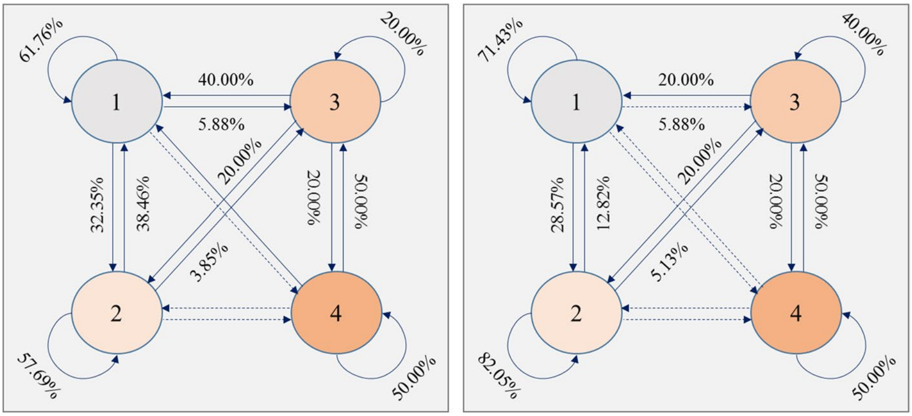

Fig. 6 Forward estimation possibility graphs of subareas C1-7 (left) and C2-8 (right)

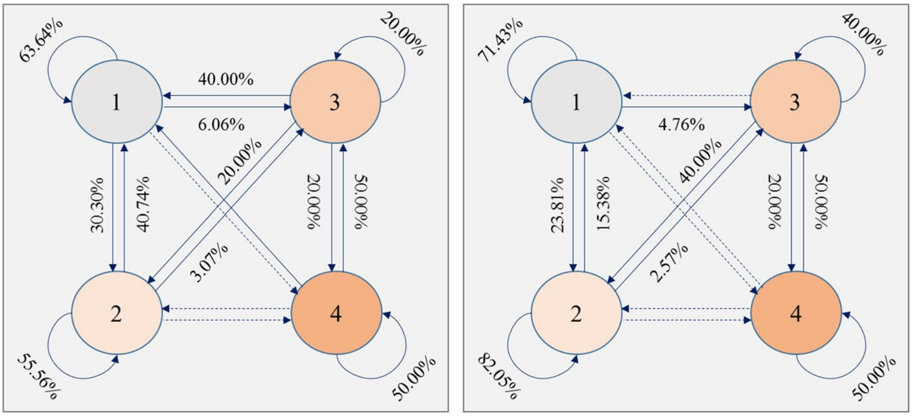

Fig. 7 Backward estimation possibility graphs for subareas C1-7 (left) and C2-8 (right)

4 Conclusions and discussion

This paper presents the use of Markov chain models in drought class transition analysis while considering the results of spatial effects relating to spatial heterogeneity and dependency. Our contributions are summarized as follows:

We firstly showed that it is unreasonable to directly apply traditional Markov chain models to analyse drought class transitions without considering the spatial effects resulting from spatial heterogeneity and dependency. We then proposed a framework for Markov chain models that considers spatial effects.

Second, three types of Markov chain models (traditional, local, and spatial) were reviewed and their application scopes were discussed. Traditional Markov chain models are suitable for analysing drought event that have no spatial effects, local Markov chain models can be used to handle spatial heterogeneity, and spatial Markov chain models can be employed to deal with spatial heterogeneity and dependency.

Third, strategies for testing for the existence of spatial heterogeneity and dependency were introduced. To test for spatial heterogeneity, spatial clustering technology was used to divide the study area into several homogenous or quasi-homogenous subareas based on the SPI time series, and the clustering validation index was then selected to identify whether spatial heterogeneity existed for drought in the case study area. To test for spatial dependency, the spatial Markov chain matrix was first constructed, and a statistical test was then applied to determine whether the local Markov chain model needed to consider the spatial context.

The proposed frameworks were used to analyse drought class transitions in Southwest China. The spatial clustering technique was used to confirm spatial heterogeneity based on the 3- and 9-month SPI time series, and different division results were obtained. However, the test for spatial dependency was unable to prove that drought states in the selected subarea depended on their spatial contexts. It is known that the spatial context plays a significant role in the test for spatial dependency. Nevertheless, in this research, spatial context was defined by the weighting value of its spatial neighbours; this involved use of a smoothing process, and it may not be capable of effectively representing particular situations (such as extreme drought in a neighbour). Additionally, a series of models have been developed to deal with the nonlinear relationships, such as a hybrid model of a Markov chain model and an artificial neural network for drought forecasting (Rezaeianzadeh et al. 2016), and a nonlinear multivariate drought index for comprehensive drought characteristic analysis (Yang et al. 2017). Nevertheless, these models also lack consideration of spatial effects.

Therefore, future research will focus on the following aspects: exploring whether drought class transitions in part by neighbours that have a drought severity or elevation that is very different from that of the study subarea; devising an expert system for drought prediction based on forward and backward estimation rules generated by local Markov chain models; and extending the proposed framework by integrating nonlinear characteristics and other information, such us Mediterranean teleconnection information (Bateni et al. 2018).

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